EDITORS' INTRODUCTION TO THE STUDENTS AT THE CENTER SERIES

Students at the Center explores the role that student-centered approaches can play to deepen learning and prepare young people to meet the demands and engage the opportunities of the 21st century. Students at the Center synthesizes existing research on key components of student-centered approaches to learning. The papers that launch this project renew attention to the importance of engaging each student in acquiring the skills, knowledge, and expertise needed for success in college and a career. Student-centered approaches to learning, while recognizing that learning is a social activity, pay particular attention to the importance of customizing education to respond to each student’s needs and interests, making use of new tools for doing so.

The broad application of student-centered approaches to learning has much in common with other education reform movements including closing the achievement gaps and providing equitable access to a high-quality education, especially for underserved youth. Student-centered approaches also align with emerging work to attain the promise and meet the demands of the Common Core State Standards. However, critical and distinct elements of student-centered approaches to learning challenge the current schooling and education paradigm:

> Embracing the student’s experience and learning theory as the starting point of education;
> Harnessing the full range of learning experiences at all times of the day, week, and year;
> Expanding and reshaping the role of the educator; and
> Determining progression based upon mastery.

Despite growing interest in student-centered approaches to learning, educators have few places to which they can turn for a comprehensive accounting of the key components of this emerging field. With funding from the Nellie Mae Education Foundation, Jobs for the Future asked nine noted research teams to synthesize existing research in order to build the knowledge base for student-centered approaches to learning and make the findings more widely available.

The topic of this paper, as with each in the series, was selected to foster a deeper, more cohesive, research-based understanding of one or more core elements of student-centered approaches to learning. The authors in this series: synthesize and analyze existing research in their areas; identify what is known and where gaps remain related to student-centered approaches to learning; and discuss implications, opportunities, and challenges for education stakeholders who put students at the center. The authors were asked to consider the above definition of student-centered approaches, but were also encouraged to add, subtract, or critique it as they wished.

The authors were not asked explicitly to address the Common Core State Standards. Nevertheless, the research proceeded as discussions of the Common Core were unfolding, and several papers draw connections with that work. The thinking, learning, and teaching required for all students to reach the promised outcomes of the Common Core provide a backdrop for this project. The introductory essay looks across this paper and its companion pieces to lift up the key findings and implications for a new phase in the country’s quest to raise achievement levels for all young people.

The nine research papers are loosely organized around three major areas of inquiry—learning theory; applying student-centered approaches; and scaling student-centered learning—although many of the papers necessarily cross more than one area:

1. LEARNING THEORY: What does foundational and emerging research, particularly in the cognitive and behavioral sciences, tell us about how students learn and about what motivates them to learn?

   Mind, Brain, and Education
   Christina Hinton, Kurt W. Fischer, Catherine Glennon

   Motivation, Engagement, and Student Voice
   Eric Toshalis, Michael J. Nakkula
2. APPLYING STUDENT-CENTERED APPROACHES: How are student-centered approaches to learning implemented? What is the nature of teaching in student-centered learning environments? How can students who are underrepresented in postsecondary education be engaged earlier and perform well in the math and reading activities that scaffold learning? How are advances in technology customizing curriculum and changing modes of learning to meet the needs of each student?

Teachers at Work—Six Exemplars of Everyday Practice
Barbara Cervone, Kathleen Cushman

Literacy Practices for African-American Male Adolescents
Alfred W. Tatum

Latino/a and Black Students and Mathematics
Rochelle Gutierrez, Sonya E. Irving

Curricular Opportunities in the Digital Age
David H. Rose, Jenna W. Gravel

3. SCALING UP STUDENT-CENTERED APPROACHES TO LEARNING: How have schools sought to increase personalization and with what outcomes for learning? What is the relationship between assessment and student-centered approaches? What can districts do to support student-centered approaches to learning?

Personalization in Schools
Susan Yonezawa, Larry McClure, Makeba Jones

Assessing Learning
Heidi Andrade, Kristen Huff, Georgia Brooke

Changing School District Practices
Ben Levin, Amanda Datnow, Nathalie Carrier

A number of distinguished researchers and practitioners serve as advisors to Students at the Center including Scott Evenbeck, founding president of the New Community College, City University of New York; Charles Fadel, Visiting Scholar, Harvard Graduate School of Education, MIT ESG/IAP, and Wharton/Penn CLO; Ronald Ferguson, Senior Lecturer in Education and Public Policy, Harvard Graduate School of Education and the Harvard Kennedy School; Louis Gomez, Professor and the John D. and Catherine T. MacArthur Foundation Chair in Digital Media and Learning, Graduate School of Education and Information Studies, UCLA; Susan Moore Johnson, Professor and the Jerome T. Murphy Professor of Education, Harvard Graduate School of Education; Jim Liebman, Simon H. Rifkind Professor of Law, Columbia University School of Law; Miren Uriarte, Professor, College of Public and Community Service, University of Massachusetts, Boston; and Arthur VanderVeen, Vice President, Business Strategy and Development at Compass Learning.

To download the papers, introductory essay, executive summaries, and additional resources, please visit the project website: www.studentsatthecenter.org.

Over the coming months, Jobs for the Future and the Nellie Mae Education Foundation will craft opportunities to engage a broad audience in the conversation sparked by these papers. We look forward to building a shared understanding and language with you for this important undertaking.

Nancy Hoffman, Adria Steinberg, Rebecca Wolfe

Jobs for the Future
Jobs for the Future identifies, develops, and promotes education and workforce strategies that expand opportunity for youth and adults who are struggling to advance in America today. In more than 200 communities across 43 states, JFF improves the pathways leading from high school to college to family-sustaining careers.

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The Nellie Mae Education Foundation is the largest charitable organization in New England that focuses exclusively on education. The Foundation supports the promotion and integration of student-centered approaches to learning at the high school level across New England. To elevate student-centered approaches, the Foundation utilizes a strategy that focuses on: developing and enhancing models of practice; reshaping education policies; increasing the body of evidenced-based knowledge about student-centered approaches and increasing public understanding and demand for high-quality educational experiences. The Foundation’s initiative and strategy areas are: District Level Systems Change; State Level Systems Change; Research and Development; and Public Understanding. Since 1998, the Foundation has distributed over $210 million in grants.

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ask a person on the street what they think of mathematics and you are unlikely to get a lukewarm answer. Some people will happily claim they love it, but more people will shudder in horror or sheepishly admit it is one of their biggest weaknesses. Mathematics conjures up memories of sitting in classrooms memorizing disconnected topics that do not seem to apply to the real world. After all, when does anyone ever use a quadratic formula? When asked about English or reading, few would answer: “I’ve never been very good at English” or “I’m just not a reading person.” But these kinds of statements are made every day about mathematics by a variety of educated people, with no sense that the statements should be questioned. As a society, we perpetuate a myth that there are just those people who are good at mathematics and those who are not. So those who do not see themselves as “math people” do not take it personally. It is just the natural order of things.

But is it?

Researchers who study the brain and the way we process numbers and concepts have shown that we are all hardwired to learn mathematics (Devlin 2001). Even babies as young as six months old can distinguish between small and large quantities (Lipton & Spelke 2003). This is true of every culture, race, and gender. It is not until we enter school that we start to see a fall-off in the number of people who do mathematics well or enjoy doing it. By the fourth grade, we see a real decline in the number of students who understand basic concepts, and this is particularly true for Latino/a and black students. So, if there is nothing wrong with our brains, no genetic predisposition, what is causing this great deficit in mathematical competency? Part of the reason may have to do with how we define mathematics and where we look for it.

This paper focuses on Latino/a and black students in the United States because of the persistent trend in these populations (especially among low-income students) for low performance on standardized tests and their lack of representation in advanced mathematics courses and mathematics-based careers. Of course, many issues make collapsing these two groups into one problematic. For example, many Latinos/as are dealing with issues of language that may not pertain to, or may play out differently for, blacks. That is, mathematics is its own language with terminology that differs from everyday use so that a student who is still learning English will have the challenge of learning two languages in the mathematics classroom. Even among Latinos/as, there are a variety of nations, races, racial phenotypes, and immigration experiences that students bring to the mathematics classroom. The same can be said of blacks, some having more recent ties to specific African, Caribbean, South American, and Asian countries, while others have ancestors who were brought over through slavery. Blacks, too, may have issues of language that racialize them differently or contribute to problems of miscommunication with teachers and others. Furthermore, class status may alter the way students receive cultural cues. There is also the fact that many Latinos/as are also black (e.g., Dominicans, Afro-Brazilians). Clearly, there is not a singular or universal Latino/a or black experience in mathematics. However, we believe the major forms of marginalization that are experienced during school by Latino/a and black youth in the United States are similar.

More so than their white peers, black and Latino/a students are strongly affected by the rigor of the mathematics curriculum, the quality of their teachers, and the beliefs teachers hold of them. For example, a review of studies asking teachers to assess the current or future ability of Latino/a and black students shows a statistically significant bias toward negative stereotypes and low expectations (Baron, Tom, & Cooper 1985). Many teachers believe the black-white achievement gap is at least partially genetic and, therefore, may be sustaining it through a self-fulfilling prophecy. In a survey of 379 secondary mathematics teachers, respondents attributed the
Students are also expected to develop a “character” that relates to mathematics. This notion of building a mathematical identity is something the research community in mathematics education takes very seriously. In fact, teachers are being asked to “empower all students to build a relationship with mathematics that is positive and is grounded in their own cultural roots and history.”
takes very seriously. In fact, teachers are being asked to “start each unit with the variety of thinking and knowledge that students bring to it” (Daro 2011), and to “empower all students to build a relationship with mathematics that is positive and is grounded in their own cultural roots and history” (NCTM 2008).

If scores on standardized achievement tests capture only a fraction of the issues we think are important in mathematical learning, where else should we turn our attention? Perspectives on mathematics as a social activity and how people learn outside of school offer a unique starting point for rethinking the problem of mathematical achievement for all students, and for Latino/a and black students in particular. This focus on student-centered learning can inform different ways of teaching and organizing schooling so that more Latino/a and black students are engaged and learn.

Focusing on mathematics that students learn outside of school (where it happens more naturally) and on mathematics applied to personal/community issues that students bring to school may better highlight the competencies and needs of these students as learners. Starting with mathematics as a social activity (as opposed to a set of skills that schools need to impart on students) may also better connect with the kinds of interdisciplinary learning that individuals will face in life.

This paper examines four categories of research:

> Ethnomathematics (e.g., cultural practices seen as unique to a particular group);
> Adults and others learning to use mathematics (e.g., for professional development in their careers; as part of their everyday practices);
> Students learning in afterschool contexts; and
> Social justice mathematics (e.g., math as a tool for addressing injustices).

The purpose of this literature review is to broaden popular conceptualizations of mathematics achievement of Latino/a and black students. By doing so, it aims to inform and inspire mathematics practitioners to craft innovative pedagogies to better support Latino/a and black youth.

We examine over eighty empirical and conceptual papers from these four categories. We privilege empirical works, distinguishing between studies that document culturally embedded uses of mathematical thinking (e.g., Kenyans crafting baskets with geometric designs) and studies that document students engaged in mathematics for a variety of purposes (e.g., in work settings or for home economics) and how engagement in these mathematical practices bears upon student achievement in school mathematics. In areas where empirical works are scarce, we draw from relevant conceptual papers to make recommendations for future research. Much of the literature comes from journal articles. However, we also considered evaluation reports and reviews of the field commissioned by foundations. This was especially true regarding afterschool learning.

We begin with early studies documenting the varied forms of mathematics that are developed and used by other cultures in their daily practices. From there, we demonstrate that “ordinary” people in the United States (e.g., carpenters, tailors, grocery shoppers) have mathematical abilities and strategies that do not transfer readily to school mathematics, in part because they are not explicitly conscious of the mathematical nature of their everyday tasks. Then we discuss the nature of mathematical learning in afterschool clubs, where novice and expert are blurred and the notion of “playing” with mathematics is more prevalent. Finally, we examine contemporary research showing that students are more engaged in using mathematics as a tool for analyzing problems when the context is injustices in society. We contrast all of this learning with the formal mathematics of schooling and suggest ways to bring the two into better collaboration.

This paper examines a wide variety of mathematical practices and competencies that are missed by school mathematics. Furthermore, it highlights the voices of learners themselves—what meanings they place on mathematics and mathematical learning.
In conducting this review, a number of questions arose:

> What causes people to do mathematics outside of school hours? What does that mathematics look like?

> Are people more competent in mathematics outside of school? If so, why?

> What keeps students from being able to apply what they know outside of school to the mathematics classroom?

> Why is it that an afterschool mathematics program that takes place at a school can look so different from a mathematics classroom in that very school?

> What kinds of social justice issues are on the forefront of students’ minds?

> Can topics in a high school mathematics curriculum (e.g., calculus) be used to explore social justice issues?
Philosophers, sociologists, and anthropologists who study mathematics have long argued that “school mathematics” is but one small version of the many forms of mathematics practiced in the world. Moreover, they have made convincing arguments that mathematics does not operate outside of individuals, morals, or power relations (Brown 1994; Clarke 2001; Ernest 1994, 2004; Fitzsimons 2002; Restivo 1994, 2007). Even mathematicians, when asked, “What is mathematics?” offer a whole host of definitions, including definitions recognizing that humans create mathematics (Burton & Morgan 2000). One needs only consider how contemporary mathematics as a research field is constantly changing and allowing for internal contradictions (e.g., catastrophe and chaos theory; undecidability; uncertainty; fuzzy logic) to see that mathematics is neither a static entity nor a field where those who practice it seek to obtain one right answer (Kline 1980). Yet, in school, we talk about mathematics in ways that ignore the fact that humans create multiple mathematics; that mathematics has a history; and that people across the globe practice it in many different ways. In some countries, mathematics is not talked about in the singular form (math) as it is in the United States; for example, it is referred to in Great Britain as “maths,” even in everyday speech. It may be that our language for talking about mathematics in the United States further engrains in students and teachers the idea that mathematics is a single entity.

Over the past two decades, research in mathematics education has moved from an emphasis on cognitive psychology (mathematics as something that happens in the minds of individuals) to mathematics in social interactions (Lerman 2000). For example, we now see knowledge as intricately tied to a person’s context, including why and with whom one is doing mathematics. From the point of view of mathematics as a social activity, teachers need to recognize it is not productive to think of their work as “teaching” students to think mathematically. Rather, teachers initiate students into mathematical communities and practices. As Sal Restivo (2007) writes:

Mathematics students might learn more effectively by recapitulating the ways the mathematical community came to collectively grasp concepts and ideas. . . . I would certainly advocate teaching mathematics in the context of their historical development. The historical, social, and cultural contexts cannot be separated from the substance of mathematical objects, concepts, and ideas.

This social perspective is so prevalent that when talking about what students “know,” many mathematics education researchers do not just consider whether students have mastered a set of predetermined procedures or facts; they also place great emphasis on identity—whether students think of themselves as people who do mathematics and how students position themselves with respect to each other in the mathematics classroom (Cobb et al. 2009; Martin 2006a, b; Esmonde & Osuna-Langer forthcoming). Given the research on teachers’ beliefs and stereotypes, issues of building strong and positive identities for Latinos/as and blacks are especially important to consider in teaching and learning.

If we consider mathematics as a social activity, where might that lead us? How does looking beyond the school walls help us better understand what mathematics students know and are doing? How can this understanding of learning outside of school help us better support Latino/a and black students in their mathematics learning in general?
Over the past three decades, a variety of research fields that have developed within mathematics education speak to the question of how people learn mathematics outside of the institution of schooling. We review here four main research areas: ethnomathematics; learning mathematics out of school and adults learning mathematics (grouped as one section because these literatures overlap to a great extent); afterschool mathematics programs; and social justice mathematics. Although similar, each of these approaches operates with assumptions and goals that have left them largely disconnected. By combining these different fields and drawing out key features, we offer a more comprehensive vision of what student-centered learning could be.

**ETHNOMATHEMATICS**

Anthropologists who study mathematics have documented that not only do all people do mathematics, but a variety of forms are practiced in different cultures. In fact, many believe that humans developed mathematics in order to describe the world around us and help us solve everyday problems. Viewing mathematics as a tool to describe the natural environment explains how very different people on different parts of the globe throughout history could create a fairly universal mathematics. And yet differences between cultures may have led to different forms being practiced. Researchers in ethnomathematics argue that the kinds of mathematics developed are partially influenced by the peoples who create them (D’Ambrosio 2006). Having developed within countries that were once colonized and that today oppose importing Western curricula, one of the primary goals within ethnomathematics is to highlight the contributions of different, mainly non-Western cultures to the field of mathematics.

At one level, this work includes documenting the mathematics that have developed throughout time (e.g., in ancient Egypt, Babylonia, India, China, and the Arab world) (Joseph 2010). However, this work also shows that indigenous peoples and adults with diverse perspectives on the world develop diverse mathematical practices (Barton 1996; Bishop 1988; Knijnik 2007; Rambane & Mashige 2007). For example, researchers studying number and pattern in South African cultures have highlighted the roles women play in reproducing geometrical patterns and tessellations through the weaving of baskets and cloth (Gerdes 1997). A common approach is for an anthropologist with extensive knowledge of mathematics to spend large amounts of time within a given population, learning how to do the mathematical work that local people do. In this sense, “ethno” refers to an identifiable cultural group (not a race or ethnicity) (D’Ambrosio 1985; 2006), such that even professional mathematicians could be seen as producing a form of ethnomathematics (Borba 1990; Powell & Frankenstein 1997).

Much of the ethnomathematics research that is readily available highlights the games that are played in many African cultures and how those games draw upon mathematical principles familiar to Western mathematics (Zaslavsky 1998; Crane 1982). One example is the set of games known as *mancala*, where stones are evenly distributed into separate cavities of a long board with two rows (resembling the base of an egg crate holding a dozen eggs). Two players alternate picking up the rocks in one cavity and distributing them equally in the successive cavities until no more rocks are left in the players’ hands. If the last rock lands in a cavity where there are other rocks, the player can continue to play with those rocks until s/he lands in a space with no rocks or home base. The goal is to start picking up rocks so as to end up with the last rock either in a cavity with a large number of other rocks or in one’s home base.
the game well requires the player to plan ahead where s/he wants the last rock to land and to also consider, among all of the permutations that could be taken, which starting point will result in the longest play and most rocks in home base.

In looking across cultures, researchers have classified the general forms of mathematics that are practiced by all humans. The most cited classification system, developed by Alan Bishop (1988), argues for six basic categories:

> **Counting** is “the use of a systematic way to compare and order discrete phenomena. It may involve tallying, or using objects or string to record, or special number words or names.”

> **Locating** is “exploring one’s spatial environment and conceptualizing and symbolizing that environment with models, diagrams, drawings, words, or other means.”

> **Measuring** is “quantifying qualities for the purpose of comparison and ordering, using objects or tokens as measuring devices with associated units or ‘measure-words.’”

> **Designing** is “creating a shape or design for an object or for any part of one’s spatial environment. It may involve making the object, as a ‘mental template,’ or symbolizing it in some conventionalized way.”

> **Playing** is “devising, and engaging in, games and pastimes, with more or less formalized rules that all players abide by.”

> **Explaining** is “finding ways to account for the existence of phenomena, be they religious, animistic, or scientific.”

Bishop’s work has a unifying sense and reminds us that at some level, mathematics is practiced in the same way across the globe.

Beyond classifying the forms of mathematics as they are practiced, ethnomathematicians also document that what the West often takes to be the exclusive knowledge of professionally trained mathematicians exists throughout the world. Among other things, researchers have shown that in the Marshall Archipelago, where sailing is integral to life and wave piloting is essential, the use of stick charts (maps) relies upon unique geometric and algebraic renderings of the oceans. The intricate designs (*kolam*) made of rice flour that are created on the threshold of a household by Tamil Nadu women of India represent transformation and superimposition of basic subunits (similar to but different from fractals). And abstract calendars used by the Maya and Balanese cultures show that not all people think of time as a linear progression in static units or as tied to the sun, moon, or other physical object (Ascher 2002). Similarly, the Xavante peoples of the Brazilian Amazon use a binary system in which units are not individual but paired (one-one or one-many), thereby challenging the deeply engrained Western belief that \(1 + 1 = 2\) (Ferreira 2001).

Few of the peoples documented in these studies have had formal schooling. Rather, they have developed these ways of using mathematics through learning from others in their community. Studies of ethnomathematics illustrate that not only do other cultures practice mathematics in sophisticated manners, but also that mathematics takes on different forms in different places. There is not one mathematics that is found everywhere in the world.

In reading this work, it may feel like ethnomathematics deals with something in the past, with primitive cultures that do not interact with the mainstream. However, that does not seem to be the case. While early studies of ethnomathematics focused on the variety of mathematical practices of diverse peoples, contemporary studies seek to highlight the asymmetrical power relations that arise when different mathematical practices are developed and maintained. For example, a recent study focused on the way a group of landless peasants in Brazil have fought to maintain the effective system they developed for measuring land plots before the school testing industry and government officials began requiring an official European system (Knijnik 2008, 2011). Testimonies by the peasants indicate that, on the one hand, doing mathematics in school denies them knowledge they have developed outside of school and are accustomed to using. It is not that they are incapable of learning the new system, but their sense of calculating the areas of land with their own system feels more connected to their roots. In this sense, it is part and parcel of their way of being and reflects the recent stance on equity taken by the National Council of Teachers of Mathematics, arguing
that mathematics should be grounded in students’ cultural roots and history (Borba 1990; NCTM 2008). On the other hand, the landless peasants’ testimonies also reflect the belief that the mathematics education they have received has not given them enough formalism and abstraction to help them negotiate a language that has traditionally kept them as outsiders.

Recent studies focusing on the perspectives of learners seem to point to the importance of learners having reference items for doing mathematics. A study of women 14 years and older in the suburbs of Brazil indicates that being able to work with familiar contexts (e.g., beans, rice, sugar) makes doing school mathematics problems easier (Fantinato 2008). Similarly, Knijnik’s landless peasants reported that concrete materials have made it easier to learn school mathematics and to teach it to others.

How does this relate to Latinos/as and blacks in the United States? For the most part, mathematics curricula rarely teach the history of mathematics—how it was developed by different peoples in different parts of the world or how it is still developing. Any history that is conveyed to students tends to be in the form of textbooks crediting mathematical theorems and discoveries to individuals of European descent (e.g., Newton, Euclid, Pythagoras, Euler, Gauss, Décartes, Fibonacci). Few students realize that the Pythagorean theorem was known by the Babylonians and Chinese more than a millennium before Pythagoras lived or that the numeral system we use today is Hindu-Arabic. Omitting this dynamic history from the classroom can give students the impression that excellence in mathematics is the exclusive domain of Europeans.

In contrast, researchers who study ethnomathematics have suggested how educators might incorporate it into school (Presmeg 1998). Some have argued that when students are connected to the things they are learning about (by introducing topics based upon cultural experiences and ideas they already have), they will be more motivated to learn (Begg 2001). This line of thinking follows a commonly held view in mathematics called “constructivism” which suggests that learning is best facilitated when new knowledge can be scaffolded onto previous knowledge; this is especially true for English learners who are in the process of learning mathematics (Gutiérrez 2002a, forthcoming; Moschkovich 2002). One design experiment showed a positive correlation between using a software tool that models tiling patterns to engage and support 18 Indian third graders in learning fractions (Sankaran 2009). The approach they use seems plausible with older students as well.

Still, others worry that a purely ethnomathematics approach (based upon the experiences that students bring with them) can turn schools into labor training institutions and possibly reinforce the subordinate position of marginalized students (Rowlands & Carson 2002). These researchers argue that problem posing and problem solving require an abstract understanding of mathematics that is not present if the starting point is always students’ experiences.

Although ethnomathematics can highlight the contributions of non-Western cultures to mathematics and the unequal power relations that arise when schools ask cultures to ignore cultural practices they have developed, a number of challenges arise for teachers interested in applying an ethnomathematics approach to their classrooms. First, in order to maintain a rigorous mathematics classroom, teachers must have a broad understanding of the history of mathematics, the identities of their students, and how the two might interrelate.

Second, there is a disjuncture between the mathematical practices that have been documented around the world and ways of relating these practices to teaching/learning. That is, few sources of lesson
plans or educational activities are available for teachers to use. This is particularly true at the middle and high school levels. We found two exceptions: activities created by the Exploratorium Museum in San Francisco; and Culturally Situated Design Tools created by Ron Eglash.

**THE EXPLORATORIUM**

One set of ethnomathematics activities available for teachers was generated and piloted by the Exploratorium (Bazin, Tamez, & Exploratorium Teacher Institute 2002). Its 14 inquiry-based activities begin with historical background on the context in which the mathematics occurs and make suggestions for how teachers can launch and assess each activity. The suggestions include such topics as: ancient Egyptian numeration, the **quipus** numerical system of the Inca, a game of solitaire from Madagascar, Mayan numeration and calendars, African **sona** (sand) drawings, and the basket-weaving patterns of many cultures. For example, the activity of **sona** drawings introduces students to the fact that in the southwest African region of the Chowke, people tell stories while drawing lines in the sand that weave in and around dots that are arranged in a rectangular array. The storyteller draws these lines without stopping, obeying rules of the manner in which the lines can weave in and out. Students are encouraged to explore how knowing the rectangular array (e.g., four by six) can predict how many closed lines (two in this case) are required to make the drawing. At an abstract level, this work involves calculating the greatest common divisor. We were unable to locate any student assessments for teachers using these activities.

Beyond having well-developed activities, incorporating ethnomathematics into school also requires that teachers know their students well and can consider their students' roots or previous experiences in an effort to link this previous knowledge with the abstract knowledge that schools require (Miranda 2008). That is, without thinking carefully about how to use activities like the **sona** drawings, teachers may inadvertently convey to their students that blacks are primitive or do not make modern-day contributions.

**CULTURALLY SITUATED DESIGN TOOLS**

Focusing on African fractals and design principles in modern culture, Eglash has developed a set of Culturally Situated Design Tools that target Latino/a, black, and Native-American students (Eglash 1999, 2010; Eglash et al. 2010). His design tools enable students to reproduce art by leveraging underlying mathematical principles in such things as: Latino-Caribbean percussion and hip-hop rhythms (ratios); graffiti (Cartesian and polar coordinates); corn-row hairstyling (transformational geometry, fractals); break dancing (rotational and sine function); and pre-Columbian architecture (symmetry, pre-algebra). Students learn about the cultural backgrounds of the art being modeled and get tutorials on how the software works before being encouraged to invent their own designs. Teachers get lesson plans, evaluation materials, suggestions for how to use the design tools, and connections to the standards of the National Council of Teachers of Mathematics.

The complexity of the designs challenges a primitive, static, or overly exotic view of culture and highlights the fact that many cultural artifacts show mathematical principles that are intentional, rather than due to individuals who are mindlessly copying others in their community. Unlike the counting systems and calendars that directly translate to Western mathematics, many urban and modern cultural practices have mathematics embedded in their processes (e.g., iteration in bead work; Eulerian paths in sand drawings). As such, they require students to create mathematical models of cultural phenomena.

Eglash has studied the impact of his culturally situated design tools on Latino/a, black, and Native-American secondary students, some of whom their teachers described as “problem students.” He found that students feel a sense of agency in creating their own designs. They also greatly improve their attitudes toward mathematics and connect social and technical domains in the creation of their identities. When given the choice to invent new designs, most students appropriate the software tools to express their identities. For example, several Puertorriqueño/a students have used the iteration program for bead work to create a Puerto Rican flag. Several low-
income black students have used the iteration program to write their initials in a way that is similar to graffiti tags. And Latino/a students have explored hip-hop music to find the least common multiple between the rotations of the rhythm wheels. In a survey of 175 randomly selected low-income eighth-grade students who have used the tools extensively, Eglash found that Latino/a and black students showed a statistically significant increase in their interest in information technology and computer-related careers; he did not find any increase among European descendent students. Moreover, three high school teachers using the tools conducted studies of their students’ learning (one or two classes per teacher) and found that students showed a statistically significant increase in grades and in pre- and post-tests in pre-algebra concepts.

SUMMARY
What we learn from these studies is that the forms of mathematics we privilege in school (e.g., Euclidian geometry; Cartesian coordinates; the base-10 counting system) are not the only mathematics that people use. And no single mathematics is produced. Moreover, the mathematical practices that have developed among different cultures serve a purpose. That is, people use mathematics not just to display knowledge to others (get good grades) as happens in school, but to accomplish something in everyday life. We also see that people learn mathematics not necessarily from someone called a teacher but also from someone in their environment who has apprenticed them into this way of using mathematics. Furthermore, individuals use mathematics in the particular ways they have learned because they make sense.

At a basic level, this research raises several questions for student-centered learning. For example, how might learning the history of mathematics and the different ways in which cultures across the world use mathematics interest students in learning more about the subject or making comparisons among its different forms? Might an emphasis on the history of mathematics (how it was created in different places at the same points in history) help them see that everyone does mathematics (including Latino/as and blacks), not just the Greeks, and that everyone changes their mathematical practices over time?

How might that affect the development of students’ mathematical identities?
How might school look if students (especially immigrants) were encouraged to use forms of mathematics they knew from their home countries?
How might students feel about themselves and their ancestors if the mathematics they knew from practices outside of school were valued or built upon? How might issues of who is the expert or novice change if community members who knew of mathematical games, weaving forms, or other practices were invited into the school to share their work and to help others learn to do it?

LEARNING MATHEMATICS: OUT OF SCHOOL AND ADULTS
What do studies conducted in North America or practices that are common there tell us about how people learn mathematics? A better understanding of the relation between mathematical use, reasoning, and motivation may be key.

Consider the typical mathematics classroom. A common question posed by students is: “When am I ever going to use this?” This question is especially pertinent for students who have not fared well in school, including Latino/a and black youth. Some students may feel that school requires them to park their identities at the door. Others may simply question a disconnected way of knowing. Regardless, the typical argument made by schools and teachers for why individuals need to learn mathematics is that the knowledge they gain is general enough to transfer to their everyday lives. Some might even say that learning mathematics helps students become critical thinkers. Teachers and textbook publishers seem to be comforted by the fact that they are creating “real-world” problems. Yet studies of people using mathematics in their work and in their everyday lives seem to challenge claims that mathematical thinking taught in schools can be applied to life or that the problems used in textbooks and mathematics lessons reflect the real world (Frankenstein 2009; Dapueto & Parenti 1999).
When adults who return to school to learn mathematics are asked why they do so, the answer is not that they seek to develop more abstract or generalized ways of using mathematics so they can apply these to their everyday working contexts. Instead, they report wanting to help their children with their homework (in ways schools expect children to represent their knowledge) or to prove to others they are smart (because mastering mathematics implies a kind of intelligence) (Wedege 2010).

Studies of people using mathematics outside of school seem to illustrate that individuals do not apply rules or ideas they have learned in the mathematics classroom to real-life problems; rather, they draw heavily on a familiar context in which they participate (e.g., Smith 2002). Poor children selling candy and melons on the streets of Brazil can calculate complex sums in the context of their work but not in similar paper-and-pencil, “school like” problems (Nunes, Schliemann, & Carraher 1993). The same has been found for adult carpet layers, grocery shoppers, interior designers, retailers, and restaurant managers (Millroy 1992; Saxe 1998, 1991; Lave 1988; Lave & Wenger 1991; Masingila 1994; Carraher et al. 1985; Schliemann 1985). For many of these individuals, the “naked” (stripped of context) problems that researchers presented to them as equivalents to what they were doing in everyday practice were seen as not equivalent at all, leading to nonsensical solutions (Carraher & Schliemann 2002). Estimating sums was easier for fourth-grade students in Italy when they could use grocery receipts because they could reason about the appropriateness of item prices with which they were familiar (Bonotto 2001). Moreover, these students reportedly could make better inferences and check the reasonableness of their answers because of the familiar context.

Looking across a variety of studies, some researchers have sought to categorize more broadly how school mathematics learning differs from mathematics learning outside of school and have identified four significant differences (Resnick 1987):

**LEARNING MATHEMATICS IN SCHOOL VS. OUTSIDE OF SCHOOL**

<table>
<thead>
<tr>
<th>IN SCHOOL</th>
<th>OUT OF SCHOOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual thinking</td>
<td>Shared thinking</td>
</tr>
<tr>
<td>Pure thought</td>
<td>Using tools</td>
</tr>
<tr>
<td>Manipulating symbols</td>
<td>Contextualized reasoning</td>
</tr>
<tr>
<td>Generalized learning</td>
<td>Situation-specific competencies</td>
</tr>
</tbody>
</table>

The distinctions in the kinds of learning are important: they point to why school mathematics does not always make sense to students or serve as a means for feeling competent. Consider this example of a mathematical exercise on “combining like terms” that students might see in an algebra course: 2x + 3y = ?.

It is not difficult to see how a student can make errors (e.g., combining unlike terms when working with variables) if manipulating algebraic symbols never involves thinking about what those symbols refer to (how each variable represents a different entity that prevents it from combining with another):

<table>
<thead>
<tr>
<th>IN SCHOOL</th>
<th>OUT OF SCHOOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x + 2y = 5xy</td>
<td>5 apples + 3 bananas = 5 apple-bananas</td>
</tr>
<tr>
<td>Could seem to make sense</td>
<td>No such thing as apple-bananas</td>
</tr>
<tr>
<td><strong>Errors persist</strong></td>
<td><strong>Reality tells us this is incorrect</strong></td>
</tr>
</tbody>
</table>

**Studies of people using mathematics outside of school seem to illustrate that individuals do not apply rules or ideas they have learned in the mathematics classroom to real-life problems; rather, they draw heavily on a familiar context in which they participate.**
Although significant reform efforts in mathematics education have been underway since these studies were reviewed in 1987, the four differences persist. Even highly technical professionals like radiologists reading x-rays or nurses calculating drug dosages use processes different from those taught in medical schools, through textbooks, or on medical rounds (Lesgold et al. 1988; Hoyles, Noss, & Pozzi 2001). Their mathematical reasoning tends to be grounded in the contexts in which they are working and in relation to others with whom they work. Other researchers have made similar claims, suggesting that the main differences between out-of-school and in-school learning are that in the former, problems are embedded in real contexts that are meaningful and therefore provide the motivation for learners to want to solve, while in the latter, the thinking processes used by learners are different from and arguably of a higher level than those taught in schools (Masingila 2002).

Studies of mathematical learning in out-of-school contexts also highlight the importance of apprenticeship (Lave 1988; Lave & Wenger 1991; Masingila 1994). Teaching in out-of-school contexts is not explicit; it is observed. Here, signs of learning are in the form not of individual acquisition of knowledge but of greater participation (performance) in the practice.

A small number of studies have looked at adolescents practicing mathematics outside of school (Masingila 2002; Nasir 2000, 2002; Nasir, Hand, & Taylor 2008; Nasir & de Royston forthcoming). Observations of black middle and high school basketball players indicate that they are competent at calculating averages and percentages for the free-throw shots of a given player when the context of the problem is a basketball game rather than a school mathematics worksheet. Furthermore, like the findings in ethnomathematics, the kinds of strategies used in the basketball context differ. For the school mathematics problem, players tend to incorrectly remember or misapply algorithms such that their strategies for finding an answer are reduced to mere manipulation of the symbols.

It is not just the strategies and understandings of mathematics that differ across the in-school and out-of-school tasks presented to the black youth; it is the differences in their sense of themselves—what they are capable of within mathematics in the different settings—that is important. This focus on identity is key for relating to the state of the field in mathematics education and for creating “character” around mathematics that is called for by the Common Core State Standards. For example, when players are asked to solve the problems in basketball contexts first, they score better on both types of problems. However, when they must solve the problems first in abstract school terms, they score lower on all of the problems. The researchers surmise that failure to solve the problems in the school context makes it difficult for players to call up more complex reasoning strategies with which they are familiar. When asked to solve the problems first in the basketball context, they seem to possess the confidence to persist in the school-based problems, even if their understanding of the algorithms is weak.

Few of the studies we located asked students for their views on learning mathematics outside of school. One exception is a study that asked 20 middle school students (10 urban, 10 suburban) to record their uses of mathematics outside of school hours in a log (Masingila 2002). Using Bishop’s six categories for analyzing their responses, this study found that students who have broadened views of what mathematics is (beyond counting, measuring, and designing) provide a greater number of examples of mathematics and include all six categories (including locating, playing, and explaining).

As such, merely asking students to take note of mathematics outside of school may not be enough to broaden their views of what mathematics is or how they practice it. A case study of adults in a folk school and prison in Finland highlights the fact that people tend to think mathematics primarily has to do with computational skills and do not see themselves as good at mathematics (Hassi, Hannula, & Nevado 2010). Similar theoretical arguments suggest that most people have very limited views of what mathematics is (Klinger 2011). The adults in these studies report that word problems aiming to model real-world situations (e.g., home, workplace, commerce) are not meaningful. One study
It seems reasonable that if adolescents are encouraged to model phenomena with which they are familiar and are expected to look for generalizations in the data they are generating, they may be more likely to learn the abstract, formal mathematics that is required in school.

Robert Moses has taken the notion of math curricula and turned it into a political issue. He asks: What is algebra? Why should students learn it? What kind of processes allow all students access to it? As a black male who had taught mathematics to his daughter and her four younger siblings, he sees the need to make a college preparatory mathematics sequence accessible to all students.

Moses recognizes that in getting students to transition between arithmetic and algebra, they must not only be able to count (number); they must also consider direction (positive versus negative numbers). Without a clear understanding of both dimensions, algebra can be confusing. As such, Moses’s Algebra Project approaches this issue by having students move through a five-step process that chronicles an event: physical event; the picture or model of the event; intuitive (idiomatic) language description of the event; a description of the event in regimented English; and symbolic representation of the event. Students do this by taking a subway trip and then mapping out their route, answering questions about “how many” and “which way.” Adolescents use this process to model phenomena from their everyday lives (e.g., cooking, painting, repairing). A key feature of the Algebra Project is beginning with where the students are and the experiences they share. Then students reflect on those experiences, draw conceptual connections to them, and finally apply that to their conceptual work:

Students learn that math is the creation of people—people working together and depending on one another. Interaction, cooperation, and group communication, therefore, are key components to this process. . . . Cooperation and participation in group activities, as well as personal responsibility for individual work, become important not only for the successful functioning of the learning group, but for the generation of instructional materials and various representations of data as well (Moses & Cobb 2001).
Evaluations of the Algebra Project indicate some success with this approach (NRC 2004). Anecdotally, the first group of students who graduated from the project enrolled in high school in geometry, and many have gone on to medical and other graduate schools. In Arkansas, 7 out of the 11 cohorts of students that were followed longitudinally showed at least a 10-point increase in mean-scaled scores on the SAT-9 a year after being in the program. Moreover, students scored at or above the proficiency level in all of the Arkansas sites, as compared with controls who declined or stayed at their proficiency levels.

Teachers who receive professional development from the Algebra Project are asked to reflect upon and address community problems. Much of the work of the Algebra Project relies upon older people who are in constant contact with a small group of youth with whom they develop meaningful relationships (Moses et al. 1989). It is unclear whether such relationships can be “scaled up.” In the words of Moses and Charles Cobb (2001):

> In the Algebra Project we have found that teachers, like students, also need nonthreatening arenas where their concerns can be articulated. . . . [T]he question remains as to whether something with that level of comfort can be institutionalized and become integral parts of school systems.

Even if scaling up might be difficult, a number of promising practices in the Algebra Project should be incorporated into more learning environments for black and Latino/a adolescents.

Some research suggests that better connecting of out-of-school and in-school practices and learning can help students:

> Prepare to deal with novel problems (both real world and non-real world); and

> Acquire the concepts and skills that are useful to solve routine everyday problems (both real world and non-real world) (Masingila 2002).

Other studies of Latino/a parents learning to use mathematics suggest that building upon students’ previous cultural experiences—what some researchers have termed “funds of knowledge”—can help address issues of equity in schools. One model is for teachers to go into the community and observe and interview families about the kinds of activities (e.g., chores) students do at home. They can build upon these forms of expertise in the classroom. However, a “funds of knowledge” approach to teaching is not simple (Civil 2002, 2007; Gonzalez et al. 2001; Moll et al. 1992). It can lead to stereotypes about particular cultural groups (e.g., presuming what kinds of experiences Latino/a adolescents bring to school) or require copious amounts of time getting to know students and their communities.

Like those who have studied ethnomathematics and shown that differences in mathematical practices create power dynamics, some researchers who study multiethnic classrooms have found that schools often ignore or even reject the knowledge that students possess from their experiences outside of school (Abreu 1999; Abreu & Cline; 2005; 2007; Abreu, Cline, & Shamsi 2002, 2000 as reported in Abreu & Cline 2007; Adler 1999; Setati et al. 2002; Setati & Moschovich forthcoming). Studying farmers and school children in a sugar cane farming community in Brazil, researchers report (Abreu & Cline 2007):

> “When farmers were exposed to modern institutions (schooling, technological innovation), this raised their awareness that some forms of knowledge were perceived as more ‘powerful’ than others.

> “Farmers passed on traditional knowledge to new generations in a selective way so that it was more likely to be passed to a child who failed at school than to a successful one.

> “Farmers’ mathematical knowledge was denied the status of ‘real’ knowledge even by children engaging in their family’s practices.

> “Farmers valued schooling and let their children attend for several years even when they failed to progress and learn.

> “School mathematics could be openly brought into farming, so that young people generated hybrid strategies and showed an understanding of how to convert between systems (e.g. of the equivalence between farming measurement and formal metrics). But this relationship was asymmetric (i.e., farming mathematics was not accepted at school).”
Follow-up questions with teachers of these students indicated that teachers presumed students would not want to become farmers. Therefore, they had little incentive to bridge the out-of-school knowledge and the in-school knowledge. Further research that this team conducted in multiethnic primary schools in England with many low-income immigrants showed similar patterns with respect to devaluing the kinds of knowledge students possessed out of school and valuing school mathematics for the kinds of careers that have high status in society.

All of these studies have been qualitative and with fairly small numbers of students. However, the ethnographic approaches (data gathered from students, teachers, and community members as well as school observations and structured tasks) and the repeated patterns across sites provide a convincing picture that when it comes to mathematics class, students are implicitly taught to ignore their out-of-school experiences. Although these studies were conducted with primary school children, the findings they report seem plausible for older students as well.

The school walls could be more permeable, with teachers taking kids into the community to study how people use mathematics in their everyday lives or inviting community members into the school to talk about the kinds of things they do and how that relates to mathematics. Then students might gain a better sense of themselves as doing mathematics and, therefore, more interested in knowing how their practices relate to formal, abstract mathematics taught in school.

Moreover, if students were encouraged to draw on their out-of-school experiences to offer multiple representations for the mathematics classroom, they might be more willing to see connections between their out-of-school mathematical practices and their in-school mathematical practices. Such approaches might also serve to position as mathematical experts students who were not previously seen as competent, based on school performance.

**SUMMARY**

Most of the studies on learning and using mathematics in everyday contexts focus on adults, not the adolescents with whom we are concerned. Not all studies indicate the ethnic or racial backgrounds of the learners or the locations of the studies. That makes it difficult to know how pertinent the findings from these studies are for Latino/a and black students. It also raises issues of how to apply this knowledge to school contexts.

Even so, findings from these studies raise important issues for teaching and learning mathematics with Latino/a and black students. For example, how might student learning differ if teachers asked students to keep a daily log of the mathematical practices outside of school in which they were engaged? How might this approach be combined with studying the histories of other cultures learning mathematics, thereby expanding students’ views of what mathematics is—and expanding the opportunities to see themselves using mathematics in their everyday lives?

How might these kinds of new school rituals help students build mathematical identities (e.g., see themselves as mathematics people)? Might students’ engagement change if teachers began with situations that were familiar, important, or in some way meaningful to their students and drew upon these funds of knowledge for launching mathematical explorations or modeling of phenomena with concrete objects?

Instead of relying upon the teacher or textbook publishers to design “real-world” problems, might mathematics learning look different if students were
encouraged to go into their own communities and find problems worth solving? Could they in some way apprentice with people in the community to learn the mathematical practices involved in the problem and then (with their teachers) build upon this knowledge to learn more formal and abstract ways of representing their solutions?

LEARNING MATHEMATICS: AFTERSCHOOL PROGRAMS

Studies on mathematics learning in afterschool settings suggest that some of the constraints normally imposed by teachers and curricula, disappear after the school bell rings. Regardless of whether or not they are on school grounds, afterschool mathematics programs can provide opportunities for greater personalization, collaboration, student talk, manipulatives, multiple representations, connections to the community, positive student identity, and, to a certain extent, more rigorous learning. Similar trends have been found in afterschool science programs led by indigenous peoples in their communities (Honey 2011).

We begin with a bird’s-eye view of national and large afterschool programs and the kinds of associated structures, content, and outcomes that are afforded to mostly low-income youth. From there, we zoom in on smaller programs that specifically connect with the lives of Latino/a and black students.

Large-scale studies and program evaluations illustrate the wide variety in focus, structure, and impact of afterschool mathematics programs (Briggs-Hale et al. 2006; Lauer et al. 2006; Mokros, Kliman, & Freeman 2005; Rothman & Henderson 2011; Welsh et al. 2002). Some programs serve as spaces for students to receive help on homework or tutoring to remediate areas of low proficiency. Others focus on preparing students for standardized tests or improving students’ attitudes toward and engagement in mathematics. Still others offer enrichment programs that can supplement what normally occurs during school hours (Mokros, Kliman, & Freeman 2005). Interestingly, when compared to other countries, the United States tends to offer more programs that focus on remediation than enrichment (Baker et al. 2001). Some afterschool programs occur on school grounds, while others are offered through community-based agencies.

A broad range of afterschool programs targets low-income youth, but the available studies consistently highlight the generally positive nature of such programs on mathematics achievement (Briggs-Hale et al. 2006; Halpern 1999; Klein & Bolus 2002; Lauer et al. 2006; Mokros, Kliman, & Freeman 2005; Rothman & Henderson 2011; Welsh 2002). One meta-analysis of 35 afterschool programs and summer schools found small but statistically significant gains in mathematics for low-achieving students (Lauer et al. 2006). The size of these gains (effect sizes of about 0.13) are meaningful when compared to those of low-achieving students who did not participate in afterschool programs, but they are insufficient to close achievement gaps between low-achieving students and their more advantaged counterparts. Offering the programs after school, during the summer, or on Saturdays does not seem to affect the program impact. However, secondary students seem to benefit more from afterschool mathematics programs than do elementary students (Lauer et al. 2006).

One possible mechanism for the increase in student achievement that is generally tied to afterschool mathematics programs is a sense of personalization. For example, programs that attend to students’ social and academic needs show greater effects than programs focused only on academics (Lauer et al. 2006). Surprisingly, even programs focused on youth development (rather than academics) improve student achievement and engagement (Eccles & Templeton 2002).

In programs focused on tutoring, one proposed mechanism for impact is the tutor-student relationship (Ritter et al. 2009). School-based teachers who work in afterschool programs can convey to students that they are important and belong to the school.

This notion of personalization extends beyond the social nature of learning (motivating students to attend) to include the number of people who can attend to students’ needs. In general, students are much more likely to work in small groups in afterschool mathematics programs than in typical mathematics classrooms and summer schools.
Moreover, programs that use small groups show greater effects than programs with whole-group instruction or a focus on one-on-one tutoring, suggesting that students may benefit from peer interaction. This focus on group work may be possible because many afterschool programs have more adults per student than is typical in mathematics classrooms (Halpern 1999). As such, adults can divide students into small groups and work with them.

A classroom culture that values what we call “horizontal learning” through peer interaction appears to go a long way toward helping students gain mathematical understanding. In contrast to “vertical learning,” where teachers convey information to students, when students can collaborate with one another to solve non-routine problems, they have more opportunities for rigorous mathematics thinking. For example, in a three-year study of how 24 black and Latino/a middle school students developed mathematical reasoning, students in an afterschool program showed increases in the variety and sophistication of reasoning with respect to proofs (Mueller 2009; Mueller & Maher 2009). The teaching and learning in this program included: open-ended, group problem solving; problems with more than one answer; encouraging students to collaborate, share, and support one another’s solutions; the use of manipulatives; and asking students to prove things that could not be proven. This contrasted with the mathematical classrooms in which the students normally participated.

In fact, this afterschool setting offered students a more relaxed environment for testing their ideas and making them public. It was a place where success was measured not by teacher approval but by peers and the reasonableness of one’s argument. Students had opportunities to hear a variety of perspectives from peers, challenge their peers in small groups and whole-class discussions, revisit their learning strategies, and refine their thinking along the way. By expanding upon the arguments of others, students in this program co-constructed proofs with more alternative forms of reasoning than if they had constructed proofs alone. They also developed greater ownership over their justifications.

A review of research on afterschool mathematics programs also suggests that when students work in small groups, they receive feedback more quickly than if they had to wait for the teacher, and they are more likely to be engaged in higher-level problem solving and making connections to the real world (Briggs-Hale et al. 2006). As such, belonging to a community of learners can offer positive results for students who are learning mathematics. Similar results for the positive influences of working in peer groups have been found with African-American students taking college-level courses, both in high schools and colleges (Fullilove & Treisman 1990).

Beyond greater personalization, dialogue, and student interaction, afterschool programs seem to offer opportunities for students to develop a kind of identity around mathematics, addressing the call from the Common Core State Standards that students create a mathematics “character.” In the study of students developing reasoning, one outcome was that students in the afterschool mathematics program later reported feeling more confident in asking questions, completing homework, and challenging the mathematical justifications of others (Mueller 2009; Mueller & Maher 2009). This sense of confidence can go a long way toward individuals’ seeing themselves as mathematical people and in persisting in solving difficult mathematics problems. In fact, some research indicates that a student’s identity within mathematics depends less on cognitive abilities and more on the kind of person the student wants to become. The form of mathematics presented to students and the kinds of community in which they are learning influences whether or not they want to become or eventually see themselves as mathematics people (Boaler & Greeno 2000).
Afterschool programs can also provide a powerful space for students to take on a different identity than they might otherwise in their mathematics classroom. One study of Patti, a third-grade black student in an afterschool, all-girls mathematics club, showed that while her mathematics classroom tended to dismiss or reject her cultural and linguistic tendencies, the club provided a space for her to express herself and do mathematics her way (Jones 2003). An analysis of observations of the mathematics club and mathematics lessons in the school, along with interviews with teachers and students over one year, suggests that part of Patti’s success in developing a mathematical identity was due to the personal relationship she developed with the club leader, a working-class white woman with cultural ties to rural Appalachia. This relationship was strikingly different from the relationship Patti had with her classroom teacher.

Within school, Patti, like many of her classmates, felt disconnected and passive. She was expected to behave and act in particular ways in order to be seen as a successful student, to do mathematics that was not connected to her personal life, and to memorize facts. In contrast, the club met once a week for 75 to 120 minutes and focused on such things as playing mathematical games (e.g., SET), identifying relationships between photography and geometrical shapes, discussing mathematics in everyday life, and taking field trips. In addition, the club invited students to be active in the room and build on the work of others. The leader of the club also worked hard to position students as competent by pointing out each member’s strengths to them individually and to the group. No student achievement data were collected, but at the end of the year, Patti claimed that she “loves mathematics.” This study, conducted with third graders, echoes studies that have associated personalization, collaboration, and manipulatives with positive outcomes for secondary students.

One multisite, afterschool research project focusing on the relationship among culture, language, and mathematics is particularly pertinent for our concerns for Latino/a youth. Situated in the Center for Mathematics Education of Latinos/Latinas, a Center for Teaching and Learning funded by the National Science Foundation, the project conducted a number of research studies at afterschool programs in the Southwest and the Midwest that sought to connect teachers, Latino/a students, and community members in doing mathematics.

At one site, Mexican middle school students and their parents did mathematics together in what was called a tertulia or mathematics circle (similar to a book discussion group). Several aspects characterized the tertulia: it was systematic; students did not attend for accreditation or promotion; attendance was voluntary; and the structure was relatively flexible and adapted to the unique situation of the participants (Menéndez & Civil 2008; Civil & Planas 2010; Diez-Palomar, Menéndez, & Civil 2011). Over two-and-one-half years and forty-two 90-minute sessions, the tertulia covered algebraic reasoning, statistics, geometry, and fractions. Typically a session would begin with mathematical learning in small groups, sometimes using manipulatives or contextualized problems, and then shift to a discussion about mathematics education. Several facilitators, including mathematics professors and undergraduate research assistants, worked with the small groups and asked participants to explain their thinking before sharing their solutions with the whole group. Because of the flexible nature of the tertulia, parents and students could suggest what kinds of mathematics they wanted to learn—whether to better understand a concept, help a child with homework, or build upon knowledge they possessed from their jobs or activities.

This format also enabled parents and students to reflect on their own mathematics learning and to comment on the instruction students were receiving in school. One complaint by parents was that schools rejected the algorithms and forms of mathematics that students or parents had learned in Mexico, thereby ignoring the “funds of knowledge” that students’ homes offered. To a certain extent, this underscores the results from other research that some Latino/a and low-income students are asked to park their identity at the door in order to participate in the mathematics classroom (Zevenbergen 2000; Gutiérrez 2002a, b). The emphasis on voluntary attendance and adapting to the unique needs of participants seems to go hand in hand with participants being intrinsically motivated to attend the sessions and engage fully in them. This study, though not reporting outcomes for participants, highlights the role of manipulatives, small groups,
and personalization, as well as maintaining a sense of cultural identity while doing mathematics.

Research at another site in this study underscores the opportunities that afterschool mathematics programs have to help students connect with their personal lives and their communities (Civil 2002). The study followed 18 third-grade and sixth-grade Mexican students at a school serving 91 percent Latino/a students, 98 percent of whom qualify for free and reduced lunch and 26 percent classified as still in the process of learning English (Diez-Palomar, Simic, & Varley 2008; Diez-Palomar, Varley, & Simic 2006; Turner, Gutierrez, & Diez-Palomar 2011). The program was non-remedial, had curricular flexibility, and focused on students’ lives. Some of the contexts for learning mathematics came from a panadería (bakery), an auto shop that converted cars into “low riders,” a nearby park that had burned down, and a dulcería that sold candies and piñatas.

Using video and audio recorders, participants observed and interviewed community members multiple times about their work and how/when they used mathematics. Then the students posed questions they wanted to answer from these contexts (e.g., how to enlarge a sketch to fit a car hood). This form of “community mathematization”—where participants collaboratively use mathematics to make sense of the world—takes problems from authentic settings in the school and community where students have personal connections. Because students are familiar with the settings, they approach problems more confidently, sometimes reenacting the practices they had witnessed by those they had interviewed.

Students who participated in this afterschool club showed increased engagement in mathematics activities; this is especially the case for those who otherwise were quiet or disruptive in mathematics classrooms that required English communication. This echoes research that afterschool settings can offer a space to construct a different identity than might otherwise occur in a school mathematics classroom. After participating in the mathematics club where group work was the norm and horizontal learning was encouraged, students were more likely to use a range of strategies to explain a mathematical operation. They also showed a greater tendency to use Spanish and work with Spanish-speaking peers during mathematics activities, suggesting that they combined identities of being Mexican with being a mathematician. However, because community members sometimes used technologies or outside sources to deal with mathematical tasks (e.g., sending their drawings to a copy center to enlarge them), some mathematical concepts remained hidden in the practice and did not translate easily to the problems that students posed. On the other hand, the club facilitators also posed problems that ignored the authentic contexts of the community in order to help students translate their understandings to more formal and abstract mathematics. Even so, the organic nature of the project made it difficult for the facilitators to anticipate mathematical connections that would arise for students.

Still another site in this larger study suggests a positive connection between participation in an afterschool mathematics program and students’ increased participation in their regular mathematics classrooms (Khisty & Willey 2011). Low-income Latino/a students in third through fifth grade who participated over a three-year period in a bilingual mathematics club had opportunities to choose and create mathematical tasks, become the authorities on problem situations, and solve problems in ways that made sense to them, using their preferred methods. Initially, students in the afterschool program were unwilling to communicate their thinking verbally or through models and drawings, but over time they learned to take more risks than in their mathematics classrooms. They also learned to ask more questions, give longer responses, use more tools to represent their thinking, and work better with others. Students also developed more positive identities around mathematics (Dominguez 2011). Undergraduate and graduate students attributed differences in the kinds of participation styles and confidence that students developed to the structure of teaching and learning facilitated. In the words of one student:

Instead of—you can’t get up on your feet in the normal class, like you have to stay with the person you are working with. You can’t

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go around and check what they’re doing to see if you or your answer . . . to see if whoever you are working with . . . to see if you got the answer right with another pair. But when you’re at the afterschool, you can move around and ask them, “Oh, what did you get? Because I got this.” And then we look at each other’s work, and we see if one of us got it wrong. And it’s kind of better than in class.

This notion—greater movement, working with peers, benefitting from the perspectives of others and building upon their ideas—is reminiscent of the work conducted with black youth in developing their reasoning abilities. It also contrasts with the kinds of teaching and learning that are typically experienced by low-income black and Latino/a youth.

Of all the approaches reviewed here, afterschool mathematics is clearly the most common. In fact, a study of 41 countries found more than one-third of all seventh and eighth graders engage in some type of out-of-school education, including cram sessions and tutoring (Baker et al. 2001). And of those who do, 4 out of 10 participate in mathematics-related activities. However, the same study found the strongest gains in mathematics achievement in nations that invest in programs focused on enrichment (e.g., South Korea, Romania) as opposed to remediation (e.g., the United States). Even so, there are tensions in the literature related to a growing overemphasis on academic gains when afterschool programs historically have served students’ social and “developmental play” needs (Halpern 2002).

Although much of the research on learning mathematics in afterschool settings points to the benefit of breaking with school traditions and supporting students in more personalized ways, some limitations in the research are worth noting. For example, few evaluation studies report the number of students dropping out of the program. As such, effects could be inflated, with the most academically motivated students staying in the program (Fashola 2002). In addition, although reports generally highlight the more personal nature of adults in these settings, some research suggests that, because of low pay and high turnover, adults in these settings may be less likely to possess deep content knowledge or familiarity with today’s mathematics curricula, especially reform-oriented formats (Halpern 1999; Mokros, Kliman, & Freeman 2005). If that is the case, recruiting better qualified staff should increase mathematical gains associated with these programs. Several successful programs targeting Latino/a youth have been run by mathematics professors, graduate assistants, undergraduate assistants, or a combination of the three.

SUMMARY

Much of the research on afterschool mathematics programs has been conducted at the middle school level. Presumably, this is a time when students are being tracked into courses that will either prepare them for college or not, and high school may be too late to significantly affect a student’s identity in mathematics or a curricular pathway. Many program evaluations were not rigorous and apparently biased toward finding positive effects, even small ones or those not directly related to achievement gains. And many of the studies collapsed students into “urban,” “at risk,” or “low-achieving” categories that made it difficult to know if students were black/Latino/a as opposed to white. To develop a solid base of evidence for student-centered learning, further research is needed to better understand the mechanisms involved in offering benefits to black and Latino/a youth, particularly through afterschool programs. For example, how do high school students feel about learning mathematics after the school day ends? What kinds of experiences do they encounter? Are their identities in afterschool programs as flexible as those of the middle school students, able to adopt a

Developing students’ confidence, enlarging their repertoire of mathematical strategies, and building a mathematical identity that builds upon one’s culture or community may be as important as increasing scores on standardized tests.
different persona than that which they embody in the school mathematics classroom? These studies seem to suggest that how students feel about themselves while doing mathematics is critical to whether or not they engage fully in mathematical activities.

The research points to a general trend among afterschool mathematics programs toward offering opportunities for personalization and other student-centered approaches, yet only smaller, qualitative studies offer voices of black and Latino/a students or their parents. And where student perspectives are available, they are mainly those of younger students. Do the same forms of personalization, collaboration, and connection to one's culture and community apply to older students? Does horizontal learning show the same kinds of benefits? How might the content of a high school mathematics curriculum influence an afterschool program's attempts to facilitate mathematical “play” or enrichment?

The research also seems to suggest important effects in terms of motivating students who might not otherwise engage in mathematics classrooms. Developing students' confidence, enlarging their repertoire of mathematical strategies, and building a mathematical identity that builds upon one's culture or community may be as important as increasing scores on standardized tests.

Several questions arise. How might everyday mathematics teachers, as opposed to professors, better incorporate community-based projects in ways that attend to the authentic nature of a community setting?

How might community agencies, as opposed to college-based researchers, leverage their resources to help provide these kinds of opportunities to more students over a longer period of time?

Can the notion of multi-generational tertulias be applied in more settings, giving parents and students more opportunities to do mathematics and reflect on their learning? Might these mathematical circles also help students better negotiate schooling? Can these community-based mathematics circles be applied in society to include not only parents and students but community members as well, especially those who might have some mathematical expertise? Might they include mathematics teachers' in ways that can help foster more positive adult-student relationships? Can they include teachers in ways that support teachers professional development on student-centered learning and transfer to the classroom?

**LEARNING MATHEMATICS AND SOCIAL JUSTICE**

One way researchers and teachers try to engage Latino/a and black adolescents who may not identify with mathematics is to begin with things that matter to them (e.g., Berry 2005, 2008; Stinson 2006, 2010). Teaching mathematics for social justice is very much like culturally relevant pedagogy, a widely embraced strategy in mathematics education in that it seeks to connect with students' out-of-school knowledge. However, teaching mathematics for social justice differs from other approaches (Leonard et al. 2010) in that students use mathematics as an analytic tool for developing an understanding and awareness of injustices in society, their place within history, and their ability to make changes in society (Frankenstein 1994, 2005; Freire 1970; Gutstein 2003, 2006).

For example, students might examine the areas and percentages of different countries on a world map to see how some countries are represented as larger or smaller than their actual land mass warrants (Gutstein 2001). Or they might survey the community on experiences with the police to calculate the likelihood that a police officer will pull over a brown or black person, versus a white person, when driving a car in a given neighborhood (Gutstein 2006). More than using mathematics as a tool for understanding social injustices that may relate to their lives, the goal is for students to also develop mathematical arguments, accompanied by representations of data that can help convince others of an action (e.g., getting a police department to rethink how it profiles drivers).

If this strategy is done correctly, students can learn mathematics by examining social and economic issues that affect their lives. In one study, elementary students received disposable cameras and took pictures of sites in their neighborhoods that were of interest to them (Leonard & Guha 2002). The students
then developed mathematics word problems based on their pictures. These held meaning for students because the contexts were familiar. Also, students had control of the kinds of questions they could ask and were personally invested in the outcomes (Martin 2006; Mukhopadhyay & Greer 2001).

Although this form of learning may sound more like what you might see in a social studies classroom, mathematics teachers have tried and succeeded with a surprising number of topics. Examples include: calculus (Staples 2005); proportional reasoning (Brantlinger 2005; Turner & Strawhun 2005; Gutstein 2003, 2006); geometry (Brantlinger 2005; Gutstein 2003); measurement (Brantlinger 2005; Turner & Strawhun 2005; Gutstein 2003); estimation (Brantlinger 2005); percentages (Diez-Palomar, Varley, & Simic 2006; Frankenstein 1990, 1995); operations with fractions (Turner & Strawhun 2005); and statistics (Gutstein 2003, 2006). Even so, in most of these studies, teachers are reporting on their own practices; few are rigorous empirical research projects with the appropriate resources to carry out adequate data collection and analysis.

Research has promoted the theory of connecting social justice issues with mathematics for some time (Frankenstein 1989, 1990; Borba & Skovsmose 1997). Yet empirical research on teaching mathematics for social justice is thin. In part, this is due to the fact that mathematics education has only recently begun to embrace issues of identity and power (Gutiérrez 2010). Also, such teaching presents additional challenges for teachers to carry out (Bartell forthcoming). In fact, it is difficult to determine the number of teachers who are implementing social justice mathematics in their classrooms: Many teachers lack the time or expertise to publish their strategies, or they simply do not think of their work as teaching mathematics for social justice because they might not create whole projects or units that embed social justice issues (Gregson forthcoming). That said, the radicalmath.org website, created in 2007 by a Brooklyn public school teacher, is dedicated to educating the public, offering resources for teaching mathematics for social justice and promoting an annual conference on the topic (Osler 2007). Of the formal studies that exist on the topic, two separate researchers and their associates, Eric Gutstein and Erin Turner, have written about their experiences. As such, we draw heavily on their work.

To illustrate, Beatriz Strawhun, a middle school mathematics teacher, worked with Turner, a college professor, to plan a six-week unit that asked students to investigate overcrowding in their middle school (Turner & Strawhun 2005). The sixth graders, predominately working-class blacks, Dominicans, and Puertoriqueños/as, expressed concern that their school, located on the fifth floor of the building, was overcrowded compared with the magnet middle school housed one floor below and serving wealthier students. After observing the implementation of the unit and related classroom discussions, the teacher and the researcher found that “as students posed problems that mattered to them, their desire to understand and affect the overcrowding increased their engagement in mathematics, and thereby enhanced the learning that occurred.” To address their own concerns about relative space, students learned ratios and proportions while practicing skills of measurement and operations on fractions. As Angel, a sixth grader in the social justice unit, noted:

> It was easier to do the math this way, instead of just learning it straight, like solving a problem, because we would actually, like, really get into it, and that made it easier. . . . Like the facts [about the school], they made you want to find out the answer. Like we wanted to know (Turner & Strawhun 2005).

The Latino/a and black students who comprised this class were motivated and engaged in solving their problem.

Turner’s students also developed a strong sense of community, taking greater risks and using one another as resources to build their solutions. Together, they constructed an understanding of the concept of ratio. The class planned to present their data to the school board and request help in improving the overcrowding situation in their school. Lianna wanted to strengthen her argument before going before the school board. In her own words, she wanted to “use more specifics so people will listen” (Turner & Strawhun 2005). When comparing the amount of space within their school to the magnet
school, Lianna did not simply calculate the square footage of the hallways for both schools. She viewed the work that a classmate was doing, comparing the amount of square footage to the number of students. She was intrigued:

“How did you do that?” she asked. “We already found out the [hallway] area of [the magnet school], and I want to see how much [space] they will each get. You found out how much each person will get in [our school], and I want to do the same thing in [the magnet school]. But I don’t know how to do it” (Turner & Strawhun 2005).

After asking for the relevant information (the number of students and total square footage), the other student, Thomas, explained to Lianna how to determine the ratio. She, in turn, presented it to the school board. By the end of the project, the class demonstrated their increased understanding of measurement, operations with fractions, and proportional reasoning as they struggled with the issue of overcrowding in their school.

Similar findings arise from the work of Gutstein, a college professor and researcher who spent one hour a day over two years teaching predominately low-income Latino/a youth in a Chicago public middle school (Gutstein 2003, 2005, 2006). In his classroom, he spent 80 to 85 percent of his time using a reform-based mathematics curriculum (Mathematics in Context), but he supplemented it with social justice projects. His goal was for his 18 students to use math to understand and change the world around them while learning and demonstrating an understanding of mathematics in the traditional schooling environment. From the point of view of Maria, one of his students, learning mathematics was more interesting in his classroom because it related to her life:

What made this experience different than other classrooms was a number of factors. First, the issues were applicable to real life, and many were personally relevant to us at more than one level. As low-income, Latino, immigrant children, some of the issues were directly linked to our own neighborhood, while others were issues of social justice on a global level (Gutstein 2006).

The engagement reported by Maria was reflective of the general class as well.

In addition, there is evidence that working from social justice contexts can benefit students mathematically. After being in Gutstein’s class for one year, students were better able to articulate their mathematical reasoning. They also passed district standardized exams for their grade level, scored better than their district peers on tests of standardized mathematics achievement, and scored well on entrance exams for competitive high schools.

Another benefit of a social justice approach is the development of a stronger sense of community in the classroom, which makes students more comfortable engaging in difficult conversations about previously taboo topics (Gutstein 2006). In addition, students become more likely to believe they can make a difference in their own lives as well as in the lives of others (Gutstein 2006; Turner & Strawhun 2005). For example, students developed more sophisticated understandings of broader social issues (e.g., using data to learn that banks were not necessarily racist, even if they tended not to loan money to blacks). From the point of view of one of Gutstein’s students:

I like the way you taught math using real life issues. That is interesting because we had never done anything like that. It got everyone thinking for themselves. It made some people come up with powerful things to say about the math involving those problems. . . . [W]e thought beyond candy, music, and soda, and it brought out another side of us. . . . All my views have changed. The world before wasn’t very interesting to me because I wasn’t aware of all the issues that were happening. Now, math made everyone interested in the real world because it’s something that catches everyone’s attention (Gutstein 2006).

Longitudinal data point to more than the immediate gains of mathematical participation, persistence with challenging concepts, and mastery of content. Learning mathematics for social justice may also raise the bar on students’ expectations for the mathematics classroom. Adrián, a former classmate of Maria, notes that after experiencing social justice
mathematics, he disengaged with mathematics when he returned to traditional instruction:

My transition to high school was difficult: It was back to the old textbook method and tedious drills. There wasn’t enough time for any critical thinking or application. My interest in mathematics decreased and my frustration grew. . . . In retrospect, the period when I learned to read and write the world with mathematics was the only time I had an interest in mathematics (Gutstein 2006).

The main contribution of teaching mathematics for social justice is that it can supplement typical mathematics curricula with topics that may be more interesting for students, increasing their engagement in mathematics. Moreover, because this approach often embeds work in the students’ local contexts, it can present problems without simple solutions and motivate students to want to understand challenging concepts (Gutstein 2006; Peterson 2005; Turner & Strawhun 2005).

Even so, this type of teaching is complex. Some of the challenges include: getting to know the students well enough to develop social justice projects that are meaningful to individuals; balancing the demands for rigorous mathematics with sufficient detail to a social justice issue; avoiding overly influencing students with the teacher’s point of view; finding time in the curriculum to fit in social justice projects; and helping students develop a sense of agency rather than despair around injustice (Bartell forthcoming; Frankenstein 1995; Freedman 2007; Gregson forthcoming; Gutstein 2006). Even mathematics teachers who work in schools with a social justice theme report these challenges.

SUMMARY

By starting with contexts that are familiar to students and appealing to their sense of fairness, teaching mathematics with social justice issues can motivate students to learn the mathematical skills necessary to solve complex problems. When presented with social justice issues, it is difficult for individuals to be indifferent; most people want to take a stand on a controversial topic. The approach appears to be especially effective at engaging students who have lost interest in mathematics, a large percentage of whom are Latino/a or black. By connecting mathematics to the world outside of school, teaching mathematics for social justice also has a way of illustrating for students that mathematics will be part of their lives after schooling.

Student voices are prominent in this line of research. They offer a consistent and convincing perspective that when learning is grounded in issues that deeply affect their lives or their communities, mathematical skills and concepts take on more meaning. The consequences for getting a “wrong answer” or having unconvincing data for an argument mean more than a poor grade. On the other hand, much of the work reported with students is anecdotal and may be influenced by students’ desires to please teachers who have chosen such methods.

The connection to social and moral development seems fairly clear, as students report being better able to understand broader social issues and articulate their stances with peers. This maturity may help position them to be more engaged citizens. Less obvious are the academic outcomes or the mechanisms by which they occur. Does learning mathematics in a social justice context offer something unique that is not present in other kinds of
meaningful contexts for students? With the exception of a few teachers reflecting on their own classrooms, none of the research focuses on high school learners. More rigorous research designs and analyses are needed to capture the kinds of learning opportunities that are afforded to students.

Some questions that come to mind if we are to take seriously a social justice framework for teaching mathematics are: How can teachers effectively and efficiently develop the knowledge of a social justice issue? Might they team up with community members who have such expertise? What might learning look like if community centers engaged schools with using mathematics to help them address some of the injustices that black and brown youth face? Could partnership projects focus on difficult social issues (e.g., violence, poverty, immigration) that would benefit society as well as student learning? Could extensive and sustained student-centered learning serve to create a generation of citizens who engage regularly with social issues?
IMPLICATIONS FOR TEACHING AND LEARNING

We began with the image of a large number of Americans who neither perform well in mathematics classrooms nor view themselves as “math people.” From there, we argued that too many of those individuals are Latino/a and black adolescents whom the institution of schooling has failed. Surveying research to understand how education in alternative contexts and modes could better support their engagement and learning, we were struck by commonalities across fields as different as out-of-school mathematics learning, ethnomathematics, adults learning mathematics, afterschool mathematics programs, and learning mathematics through social justice issues. We report on the commonalities in the form of recommendations to teachers, policymakers, and funders.

BUILD UPON FAMILIAR CONTEXTS AND THE PERSONAL AND CULTURAL EXPERIENCES OF LEARNERS

Schools tend to ignore or even reject the familiar contexts and personal and cultural experiences of learners. Any connection to the real world tends to come from word problems developed by textbook publishers or teachers; it is not clear that learners find these meaningful. Student-centered learning for Latino/a and black adolescents would seek to build upon their experiences in ways that help position individuals as “experts” with something to share. Some examples that learners could be encouraged to draw upon while engaging in mathematics might include known games, algorithms from other countries, and hobbies or community practices. It is worth noting that in programs that have built upon familiar contexts and cultural experiences, typically members of the communities have been facilitators who took the time to get to know students in deeper ways. Rather than relying upon stereotypes or their own impressions, educators would need to create space and time for learners to inform them about which contexts are familiar and which cultural experiences are meaningful. Joint community walks, projects that allow for students to apply their lives, and more personal conversations with students can help create more student-centered learning experiences for black and Latino/a adolescents.

NURTURE CONFIDENCE AND A MATHEMATICAL IDENTITY IN LEARNERS

Secondary mathematics classrooms tend to focus on mastering predetermined content, with little attention to students’ social or emotional development. Yet black and Latino/a adolescents, like others, seem to reap the benefits of programs that attend to both their academic and social needs. Almost at the flip of a switch, they can turn from passive, disruptive, disengaged students into learners who are full of...
energy, offer creative strategies, and have a desire to discuss mathematics with others. In fact, learners show greater confidence and abilities to not only arrive at an answer but also reflect on how reasonable that answer is when they have opportunities to: be active in a learning space; use their home languages; build upon familiar contexts and personal and cultural experiences; use mathematics to analyze injustices in society; and apply strategies that make sense to them. However, learners need help translating their everyday knowledge into more abstract forms of mathematical modeling and representation. Manipulatives and community members, especially ones from a familiar context, can serve as useful tools to make this translation.

Findings across the research areas suggest that an individual’s identity is tied to the practices he/she has created, regardless of whether the learner is a member of a landless peasant movement, a parent who wants to help a child with homework, or a teenager who wants to do things her own way. Incorporating the history of mathematics and the views of community members can go a long way toward helping students see that mathematics is not a singular entity, that many cultures have created (and are still creating) it, and that we can combine our personal identities with mathematical ones. Creating opportunities for students to have a stronger voice in the kinds of mathematics being studied and the forms of interaction in a learning environment can also help black and Latino/a learners see themselves as “math people.” And when they see themselves as such, they are more likely to persist in solving difficult problems or addressing novel situations.

**USE AUTHENTIC PROBLEMS AND OTHER LEARNERS TO INCREASE MATHEMATICAL RIGOR**

For decades, mathematics problems have tended to be of the kind: “Here is an example where I have worked out a solution, now you do 30 of them.” In fact, these are not problems but mere exercises. The teacher and the students both know there is only one right answer, and probably only one sanctioned way of representing the solution. In fact, the latest push—for failing schools to better prepare students for standardized exams—almost ensures that Latino/a and black youth will continue to get this form of instruction. In contrast, problems in the real world involve many overlapping variables and are not so clear cut. They require learners to decide what information is pertinent to the problem at hand, what strategy might best fit the situation, whether an exact answer or a good approximation is warranted, and how best to test that strategy in practice with others. Almost never is there one right answer, with a predetermined set of procedures to be followed.

By beginning with problems that are grounded in Latino/a and black students’ interests in the world, we increase the chances that they will be engaged in higher-order thinking. Such problems invite learners to bring previously acquired knowledge to the table to help in deciding how best to develop a solution. This may be especially true for broader social issues that affect black and Latino/a adolescents, such as social injustices that motivate them to find answers to their questions.

The potential for higher-order learning is further pronounced when individuals work with peers in horizontal learning structures. Just as in real life, where collaboration often requires greater levels of energy and attention to detail, so, too, can small-group problem sessions require more of learners. If structured around an authentic, open-ended problem, Latino/a and black adolescents can benefit from working in small groups. They can hear a variety of perspectives and strategies, refine their thinking, and represent and justify their ideas to others. In doing so, they are more likely to persist in a trajectory from novice to apprentice to expert.

**LEVERAGE COMMUNITY MEMBERS TO ADD PERSONALIZATION AND CHALLENGE STATIC NOTIONS OF “NOVICE” AND “EXPERT”**

Schools operate under the idea that one mathematics teacher can effectively support 25 to 30 students with largely whole-class instruction. This organization presumes the teacher needs few opportunities to
understand deeply what individual students know or can do—both before they walk into the classroom and after they walk out of it. However, even when teachers choose to work in small groups, they and their students generally benefit more when they have time to check in with one another about the status of their work, their growth, and their misconceptions. We take our lead from projects that had favorable facilitator-to-student ratios and suggest asking community elders to work in schools.

By bringing in elders who may be unfamiliar with today’s mathematics curriculum, classroom manipulatives, or mathematical technology to teach mathematics (e.g., Geometer’s Sketchpad), notions of authority shift. Adolescent learners can “teach” peers and adults about things with which they are familiar, and also learn from/with individuals who have a lifetime knowledge of the real world and how mathematics may relate to it. This blurring of who is novice and who is expert can go a long way toward developing meaningful personal relationships, while offering opportunities for students to try on different identities.

We have offered some of the ways that teachers might adopt strategies found in out-of-school settings. However, the point of placing students at the center of learning is not to take all of the components of learning that have occurred outside of school hours and squeeze them into mathematics classrooms.

Schools, as institutions, are constrained, among other things, by their organizational structures, goals, and teacher credentialing processes. One of the greatest tensions for secondary mathematics teachers is attending to issues of depth versus breadth: Do I move on with tomorrow’s topic if not everyone understands today’s, or do I sacrifice time on the next topic in order to develop greater understanding of this one?

Teachers and schools organize their work (and subsequent student learning) based upon how much of a prescribed curriculum can be covered in the amount of time that is available during the school day. In the case of today’s schooling, whole-class delivery, standardized assessments, and multiple sections of the same course offer few options for mathematics teachers other than to either move on with the whole class or keep the whole class focused on the topic for a longer period.

In contrast, the interdisciplinary nature of life, the desires of individuals and communities, and the assessment of (and consequences for) successful problem solving all drive a very different process in learning outside of school or in situations that interface with social and community issues. To create more opportunities for student-centered learning, we must think differently about the enterprise of education—where and when it happens, and who benefits from its forms.

Adolescent learners can “teach” peers and adults about things with which they are familiar, and also learn from/with individuals who have a lifetime knowledge of the real world and how mathematics may relate to it. This blurring of who is novice and who is expert can go a long way toward developing meaningful personal relationships, while offering opportunities for students to try on different identities.
Although the research gives us a starting point for building student-centered approaches to improving mathematics learning for Latino/a and black youth, there is much we cannot learn from the literature. First, little empirical research centers directly on the learning of U.S. Latino/a and black adolescents. This is particularly true for the topics of ethnomathematics and adults learning mathematics. As such, it is difficult to know how applicable the research findings might be for such populations. Moreover, even research focused on black or Latino/a adolescents does not necessarily ask for the learner’s perspective. Thus, some “findings” may be biased by researchers, many of whom are not black or Latino/a themselves.

If the research reviewed is not always specific to Latino/a and black youth, how might our conclusions significantly improve learning conditions for those adolescents? In some ways, our recommendations can be viewed as simply “good teaching”—that is, good for all students and not particular to black or Latino/a youth. However, we return to the history of mathematics teaching and learning for marginalized students and highlight the fact that black and Latino/a adolescents tend to have low-quality mathematics and teachers who see their failure as related to student motivation or family background. Perhaps more important for the development of a strong and positive identity around mathematics, blacks and Latinos/as rarely have opportunities to bring their culture, language, previous experiences, or sense of justice to the mathematics classroom. As such, incorporating out-of-school experiences can help them maintain a sense of self that is whole, rather than requiring them to “park their identity at the door.” Furthermore, having Latino/a and black youth bring their lives and a sense of social justice into the learning of mathematics may also enrich learning for white and Asian students who may be unaware of the contributions of Latinos/as or blacks or who may hold stereotypes about who is capable of doing mathematics.

Although we highlight some interesting activities taking place in out-of-school contexts, much of the work in community-based organizations and settings is not documented in the literature because such programs do not require formal evaluations or because community members are busy engaging in the work, leaving little time to write about it. As such, we have little understanding of efforts that are more organic or that involve volunteers from the community. These programs fly under the radar of research. In addition, programs serving undocumented students (e.g., Latino/a immigrants) or students who are seen as vulnerable according to the guidelines of Human Subject Review Boards (e.g., low-income students, homeless individuals) may have a harder time getting permission from participants or their families to study the structure or outcomes of their programs. Furthermore, these populations may be less likely to voice their opinions to researchers, not wanting to call attention to themselves or their families. Greater care and trust need to develop across schools, neighborhoods, and community-based institutions (e.g., churches, Boys & Girls Clubs) so that the rich knowledge that they possess about learners can be tapped. In addition to helping adolescents develop socially, explicit connections need to be made with doing mathematics so that people of all ages view this social activity as normal and enjoyable.

Overall, there is a lack of longitudinal data. Few studies report on more than three years of work with learners. Most of the studies that follow participants for more than one year are working with fewer than 20 students. As such, it is hard to know how different programs and experiences influence students over the long term—either with respect to how they view themselves or how well they can do mathematics.
Perhaps the positive effects we see on engagement and mathematics achievement through afterschool programs, using social justice issues, or justifying their answer to peers quickly wear off in less learner-centered environments. Or positive effects might blossom later in life as adolescents mature or graduate from compulsory education that constrains their ways of interacting.

From looking outside of school to understand how people use and learn mathematics, there is still very much we do not know. Many forms of mathematics are simply not well studied or are somewhat hidden by the technology employed (e.g., video gaming). Some of the most interesting research we found (e.g., community-based mathematics discussion groups) was conducted by skilled mathematics professors and their college students as research assistants in settings with fairly favorable facilitator-to-learner ratios. It remains to be seen whether such efforts can be carried out with less formally educated facilitators, with high school aged students, or without the resources of a research grant. The same could be said of teaching mathematics for social justice or teaching. Most of the teachers applying this approach are full-time college faculty members who choose to teach one public school mathematics class a day for research purposes. Generally, they are not juggling several sections of mathematics with 200 students or more—or feeling pressure to “teach to the tests” like full-time teachers who serve black and Latino/a youth. And teachers who seem to be applying principles of personalization effectively, while helping students transition between everyday life and the symbolic forms of mathematics valued in society, have been well supported to develop their expertise. Pedagogical approaches that build upon the history of mathematics or students’ cultures require a high level of cultural proficiency and a very broad understanding of mathematics—historically, culturally, and practically.

We need to take the most successful projects and scale them up, following them for longer periods, so more black and Latino/a adolescents can benefit. For approaches and projects that offer promise, we need to develop more rigorous assessments, invite students of varying ages, and cover a broader range of mathematical topics in order to better understand which formats best serve which purposes.

If we are to take seriously the idea of placing black and Latino/a students at the center of learning, we must engage the broader public in the endeavor, especially community-based organizations that have vested interests in supporting youth. We might also take our guidance from countries like Cuba that have embedded the notion of education as a social responsibility of all citizens, where widespread literacy has become a national goal. In much of Cuba, the walls separating communities, businesses, schools, adults, children, learners, and educators do not exist or are much more permeable than in the United States. Spain is another country that offers a way of thinking about learning centered on students. Using a model called comunidades de aprendizaje (learning communities), hundreds of Barcelona schools leverage community resources, technology, schools, and modes of regular and intergenerational dialogue to translate abandoned and low-income neighborhoods into vibrant places with increased mathematics achievements for youth, greater ownership and strong community, and lifelong learning opportunities for adults. We need to explore countries like these that have taken a strong, comprehensive stand on learning.

Comunidades de Aprendizaje

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Descriptions of the variety, complexity, and personal meaning involved when people use and create mathematics to solve everyday problems outside of school hours point to the outdated mode of thinking in today’s mathematics classroom, where the emphasis is on working alone, ignoring the contexts in which mathematical problems arise, and privileging the use of symbolic representations before they have any significance to learners. What remains to be seen is whether, as a nation, we have the courage to build on this knowledge base to make important decisions about how we will move forward in this enterprise we call mathematics education.
ENDNOTES

1 See: http://www.studentsatthecenter.org/papers/personalization-schools

2 See: http://www.studentsatthecenter.org/papers/literacy-practices


4 See: http://www.studentsatthecenter.org/papers/teachers-work

5 See: http://utopiadream.info/ca


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